Both residual errors accurate algorithm for inverting general tridiagonal matrices

Paweł Keller and Iwona Wróbel

Warsaw University of Technology, Poland

Abstract

Even though in most problems involving matrix inverse the numerical computation of the actual inverse is usually not necessary (the problem may be reformulated to solve a corresponding system of linear equations or a corresponding matrix equation), there seems to exist no computational system or numerical library which would miss a subroutine for numerical computation of the matrix inverse.

When using such a subroutine one could expect to obtain the most accurate result possible. Unfortunately, all numerical algorithms (that are known to the authors) for computing the matrix inverse suffer a *curse* that the larger of the residual errors, $\|AX - I\|$ and $\|XA - I\|$ (X denotes the computed inverse of a matrix A), may grow as fast as $\text{cond}^2(A)$, where $\text{cond}(A)$ is the condition number of A (we assume that A is not a triangular matrix).

In our presentation, we present the algorithm for inverting general tridiagonal matrices that overcomes the above curse, i.e. it computes the inverse for which both residual errors grow linearly with $\text{cond}(A)$. In addition, the proposed algorithm has the smallest possible asymptotic complexity for the considered problem.

The proposed method is based on careful selection of formulas for the elements of A^{-1}, which preserves all recursive properties resulting from the equations $AX = I = XA$. Extensive numerical tests confirm very good numerical properties of the algorithm.

Keywords

Matrix inversion, Tridiagonal matrix, Recursive algorithm, Numerical stability.