Root location of polynomials with totally nonnegative Hurwitz matrix

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Abstract

For a given real polynomial

$$p(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n, \qquad a_0 > 0,$$

the $n \times n$ matrix $H_n(p) = (a_{2j-i})$ is called finite Hurwitz matrix, and the matrix $\mathcal{H}_\infty(p) = (a_{2j-i})_{i,j \in \mathbb{Z}}$ is the *infinite* Hurwitz matrix.

It is known [3, 2] that the total positivity of the matrix $\mathcal{H}_\infty(p)$ is equivalent to stability of the polynomial $p(z)$ (roots in the open left half-plane), while the totally nonnegativity of [1, 4] the finite Hurwitz matrix $H_n(p)$ does not imply stability of $p(z)$. In this talk, we completely describe root location of the polynomial $p(z)$ whose finite Hurwitz matrix $H_n(p)$ is totally nonnegative.

Keywords

Root location, Polynomials, Totally nonnegative matrices.

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