Immanant inequalities on correlation matrices and Littlewood-Richardson's correspondence

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Abstract

The Littlewood-Richardson rule is one of the most important properties to describe the representation theory of the symmetric group, i.e. the coefficient of the product of Schur functions can be calculated in a combinatorial way using Young diagrams. In [1], it is also pointed out that immanants, which are special cases of generalized matrix functions labeled by Young diagrams, have the same rule.

One of the most famous open problems involving immanants is Lieb's permanental dominance conjecture ([3]), a sort of analogue of Schur's inequalities ([4]). In this talk, we analyze the correlation matrix $Y_n = (n/(n-1)\delta_{ij} - 1/(n-1))$, which conjecturally gives sharper bounds of the inequalities, where δ_{ij} is the Kronecker delta function. Motivated by Frenzen-Fischer's result ([2]), i.e. $\lim_{n\to\infty} \operatorname{per} Y_n = e/2$, we explore the limiting behavior of immanants through the limit shape of Young diagrams. The Littlewood-Richardson rule will be applied as the key lemma. We will introduce other related topics.

Keywords

Immanant, Symmetric group, Littlewood-Richardson rule.

References

[1] D. E. Littlewood, A. R. Richardson (1934). Group Characters and Algebra, *Philos. Trans. R. Soc. Lond. Ser. A, Math. Phys. 233*, 99–141.

[2] C. L. Frenzen, I. Fischer (1993), On a conjecture of Pierce for permanents of singular correlation matrices, SIAM J. Matrix Anal. Appl. 14, 74–81.

[3] E. H. Lieb (1966), Proofs of some conjectures on permanents, *I. Math. and Mech. 16*, 127–134.

[4] I. Schur (1918), Über endliche Gruppen und Hermitische Formen, *Math. Z. 1*, 184–207.