More about linear sufficiency in the linear mixed model

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Abstract

A linear statistic $\mathbf{Fy}$ is called linearly sufficient, or shortly BLUE-sufficient, for the estimable parametric function of $\mathbf{K}\beta$ under the linear model $M = \{\mathbf{y}, \mathbf{X}\beta, \mathbf{V}\}$ if there exists a matrix $\mathbf{A}$ such that $\mathbf{AFy}$ is the best linear unbiased estimator, BLUE, for $\mathbf{K}\beta$. Similarly, $\mathbf{Fy}$ is called linearly prediction sufficient, or shortly BLUP-sufficient, for the new "future" observation $\mathbf{y}_*$, say, if there exists a matrix $\mathbf{A}$ such that $\mathbf{AFy}$ is the best linear unbiased predictor, BLUP, for $\mathbf{y}_*$. The new observation $\mathbf{y}_*$ is satisfying $\mathbf{y}_* = \mathbf{X}_*\beta + \mathbf{e}_*$, where $\mathbf{X}_*\beta$ is estimable, and the covariance matrix between $\mathbf{e}_*$ and $\mathbf{y}$ is known. Our purpose is to predict $\mathbf{y}_*$ on the basis of $\mathbf{y}$.

The concept of linear sufficiency was essentially introduced in early 1980s by [1, 2]. In this paper/talk we generalize their results in the spirit of recent papers [3] and [4]. In particular, we pay attention to the linear sufficiency of $\mathbf{Fy}$ with respect to $\mathbf{y}_*$, $\mathbf{X}_*\beta$ and $\mathbf{e}_*$ and the mutual relations between these sufficiencies.

Keywords

BLUE, BLUP, Linear sufficiency, Linear mixed model.

References

[1] Baksalary, J.K. & Kala, R. (1981). Linear transformations preserving best linear unbiased estimators in a general Gauss–Markoff model. *The Annals of Statistics*, 9:913–916.

[2] Baksalary, J.K. & Kala, R. (1986). Linear sufficiency with respect to a given vector of parametric functions. *Journal of Statistical Planning and Inference*, 14:331–338.

[3] Kala, R., Puntanen, S. & Tian, Y. (2017). Some notes on linear sufficiency. *Statistical Papers*, 58:1–17.

[4] Markiewicz, A. & Puntanen, S. (2017). Further properties of the linear sufficiency in the partitioned linear model. *Matrices, Statistics and Big Data*, Proceedings of the 25th International Workshop on Matrices and Statistics, IWMS-2016, Funchal, Madeira, Portugal, 6–9 June 2016. Springer, in press.