Neglecting non-diagonalizable matrices in social sciences

Pieter-Jan Pauwelyn and Marie-Anne Guerry

Vrije University Brussels, Belgium

Abstract

In social sciences, many transition processes are described by Markov models[1]. Markov chains are characterized by stochastic matrices. In this paper, the interest lies with the non-diagonalizable stochastic matrices. We will explicitly show that it is possible for every non-diagonalizable stochastic 3×3 matrix to be perturbed into a diagonalizable stochastic matrix with real eigenvalues arbitrarily close to the original eigenvalues. This is done by using an additive perturbation[3]. This is based on the denseness of diagonalizable matrices in the set of stochastic matrices[2]. Moreover, every non-diagonalizable stochastic 3×3 matrix can be perturbed into a diagonalizable stochastic 3×3 matrix such that the principal (left and right) eigenspaces of the original matrix and the perturbed matrix coincide. An algorithm is presented to determine the perturbation matrix. Finally, a theorem is proved which shows that there are even more parallels between the original matrix and the perturbed matrix.

Keywords

Markov models, Stochastic matrices, Non-diagonalizable matrices, Perturbation theory.

References

[1] Bartholomew, D.J. (1973).*Stochastic Models for Social Processes.* Wiley, Chichester.

[2] Hartfiel, D.J. (1995). Dense sets of diagonalizable matrices. *Proceedings of the American Mathematical Society 123*, 1669–1672.

[3] Stewart, D.W. and Sun, J. (1985). *Matrix Perturbation Theory.* Academic Press, New York.