Kronecker product approximation via entropy loss function

Katarzyna Filipiak[1], Daniel Klein[2], Augustyn Markiewicz[3], and Monika Mokrzycka[4]

[1] *Poznań University of Technology, Poland*
[2] *P. J. Šafárik University, Košice, Slovakia*
[3] *Poznań University of Life Sciences, Poland*
[4] *Technical Communication College, Poznań, Poland*

Abstract

The aim of this talk is to determine the best approximation of a positive definite symmetric matrix $\mathbf{\Omega}$ of order n by $\mathbf{\Psi} \otimes \mathbf{\Sigma}$, where square matrices $\mathbf{\Psi}$ and $\mathbf{\Sigma}$ are arbitrary (unstructured) or one of them, say $\mathbf{\Psi}$, can be structured as compound symmetry (CS) correlation, i.e., $(1-\varrho)\mathbf{I} + \varrho\mathbf{1}\mathbf{1}^\top$, or autoregression of order one (AR(1))) correlation, i.e., $\sum_{i=0} \varrho^i \left(\mathbf{C}^i + \mathbf{C}^{i\top}\right)$ with $\mathbf{C} = (c_{ij})$, and $c_{ij} = 1$ if $j - i = 1$ and 0 otherwise. The best approximation means here that the entropy loss function (cf. [1])

$$f(\mathbf{\Omega}, \mathbf{\Psi} \otimes \mathbf{\Sigma}) = \operatorname{tr}\left(\mathbf{\Omega}^{-1}(\mathbf{\Psi} \otimes \mathbf{\Sigma})\right) - \ln\left|\mathbf{\Omega}^{-1}(\mathbf{\Psi} \otimes \mathbf{\Sigma})\right| - n$$

is minimized with respect to $\mathbf{\Psi} \otimes \mathbf{\Sigma}$, where $\mathbf{\Psi}$ is unstructured or structured as CS or AR(1).

We show that for a given $\mathbf{\Omega}$ and positive definite component of $\mathbf{\Psi} \otimes \mathbf{\Sigma}$, say $\mathbf{\Sigma}$, the best approximation is obtained for positive definite $\mathbf{\Psi}$.

Presented results can be widely used in multivariate statistics, for example for regularizing the covariance structure of a given covariance matrix, for determining the estimators of covariance structure or for testing hypotheses about the covariance structures.

Keywords

Kronecker product, Approximation, Entropy loss function.

References

[1] Lin, L., N.J. Higham, and J. Pan (2014). Covariance structure regularization via entropy loss function. *Comput. Statist. Data Anal. 72*, 315–327.