Confidence Regions and Tests for Normal Models with Orthogonal Block Structure: Pivot Variables

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Abstract

Models with Orthogonal Block Structure, OBS, have variance covariance matrices that are linear combinations of pairwise orthogonal orthogonal projection matrices that add up to $\boldsymbol{I}_n$,

$$\boldsymbol{V}(\boldsymbol{\gamma}) = \sum_{j=1}^{m} \gamma_j \boldsymbol{Q}_j, \tag{1}$$

see [1] and [2]. These models continue to play an important part in the theory of randomized block designs and contain the models

$$\boldsymbol{Y} = \boldsymbol{X}_0\boldsymbol{\beta} + \sum_{i=1}^{w} \boldsymbol{X}_i \boldsymbol{Z}_i, \tag{2}$$

where $\boldsymbol{\beta}$ is fixed and the $\boldsymbol{Z}_1, ..., \boldsymbol{Z}_w$ are independent, with null mean vectors and variance covariance matrices $\boldsymbol{\sigma}_i^2 \boldsymbol{I}_{c_i}, i = 1, ..., w$, when the matrices $\boldsymbol{M}_i = \boldsymbol{X}_i \boldsymbol{X}_i^\top$ commute and $R([\boldsymbol{X}_1, ..., \boldsymbol{X}_w]) = R^n$. We will assume normality to use pivot variables to obtain confidence regions and, through duality, test hypothesis both for:
-variance components $\gamma_1, ..., \gamma_m$ and $\sigma_1^2, ..., \sigma_w^2$;
-estimable functions $\psi = \boldsymbol{c}^\top \boldsymbol{\beta}$ and estimable vectors $\boldsymbol{\psi} = \boldsymbol{C}\boldsymbol{\beta}$.

In deriving confidence regions for the $\sigma_1^2, ..., \sigma_w^2$ and ψ we had to apply the Glivenko-Cantelli theorem and related results to samples of values of pivot variables. Moreover, for $\boldsymbol{\psi}$, we had to consider families of samples in order to adjust confidence ellipsoids using a technique similar to least square adjustment of linear regression.

We include a numerical application to the results of an grapevine experiment. This application is interesting in showing the good behaviour of pairs of samples for the positive and negative parts of the $\sigma_i^2, i = 1, ..., w$. Then we show that we have $\sigma_i^2 = \sigma_i^{2^+} - \sigma_i^{2^-}$, with $\sigma_i^{2^+}$ and $\sigma_i^{2^-}$ linear combinations of the $\gamma_1, ..., \gamma_m$.

Keywords Confidence Regions, Pivot Variables, UMVUE, Variance components

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