Linear spaces of symmetric nilpotent matrices

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Abstract

In 1958 Gerstenhaber showed that if $\mathcal{L}$ is a subspace of the vector space of the square matrices of order n over some field $\mathbb{F}$, consisting of nilpotent matrices, and the field $\mathbb{F}$ is sufficiently large, then the maximal dimension of $\mathcal{L}$ is $\frac{n(n-1)}{2}$, and if this dimension is attained, then the space $\mathcal{L}$ is triangularizable. Linear spaces of symmetric matrices seem to be first studied by Meshulam in 1989 in view of the bound of their rank. Although it seems unnatural to ask when a linear space of symmetric matrices is made of nilpotents and when it is triangualar, we find a way to do so by going to an equivalent notion for symmetric matrices, i.e. persymmetric matrices. We develop a theory that enables us to prove extensions of some beautiful classical triangularizability results to the case of symmetric matrices. Not only the Gerstenhaber's result, but also Engel, Jacobson and Radjavi theorems can be extended. We also study maximal linear spaces of symmetric nilpotents of smaller dimension.