Tightening bounds on the radius of nonsingularity

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Abstract

Evaluating the proximity of a given square matrix to the nearest singular one can be performed via adopting Chebyshev norm leading to so called radius of nonsingularity. Let A be a a matrix of a form $\mathbb{R}^{n\times n}$ and Δ is non-negative matrix of the same type, the *radius of nonsingularity* [1, 2] is defined by

$$d(A,\Delta) := \inf\{\varepsilon > 0;\ (\exists \text{ singular } B)(\forall i,j) : |a_{ij} - b_{ij}| \le \varepsilon\Delta_{i,j}\}.$$

There also exists a simplified version of such radius where Δ is equal to "all ones matrix" E. Determining exact value of this radius even in its simplified version is known to be an NP-hard problem [2], which leads to various lower and upper bounds [3, 4]. These bounds, however, are not very tight - one of the best classical bounds has the relative error $6n$. We describe a better one based on a randomized approximation method with expected error 0.7834 using a semidefinite relaxation [5] and discuss its possible extensions depending on various conditions given.

Keywords

Radius of non-singularity, Regularity, Interval matrix, Bounds

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