Properties of partial trace and block trace operators of partitioned matrices

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Abstract

The aim of this paper is to give the properties of two linear operators defined on $pq \times mq$ partitioned matrix $\mathbf{A} = (\mathbf{A}_{ij})$ with $q \times q$ blocks $\mathbf{A}_{ij}$:

- partial trace operator, $\mathrm{PTr}_q\mathbf{A} = (\mathrm{tr}\mathbf{A}_{ij})$ (cf. [1]), and
- block trace operator for $m = p$, $\mathrm{BTr}_q\mathbf{A} = \sum_{i=1}^{p} \mathrm{tr}\mathbf{A}_{ii}$.

The conditions for symmetry, nonnegativity, and positive-definiteness of the operators are given, as well as the relations between partial trace and block trace operators with standard trace, vectorizing and Kronecker product operators.

Both partial trace and block trace operators can be widely used in statistics, for example in the estimation of unknown parameters under the multi-level multivariate models or in the theory of experiments for determination of optimal designs under linear models.

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References

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