Diagonal Elements in the Nonnegative Inverse Eigenvalue Problem

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Abstract

We say that a list of complex numbers is *realisable* if it is the spectrum of some (entrywise) nonnegative matrix. The Nonnegative Inverse Eigenvalue Problem (NIEP) is the problem of characterising all realisable lists.

Although the NIEP remains unsolved, it has been solved in certain cases. In particular, the solution is known if the list contains at most three elements or if every entry in the list (apart from the Perron eigenvalue) has nonpositive real part. In these cases, if a realising matrix is known to exist, one may ask what the possible diagonal elements of said matrix are. For a given realisable spectrum, we show that a list of nonnegative numbers may arise as the diagonal elements of the realising matrix if and only if these numbers satisfy a remarkably simple inequality. The realising matrices employed are of a similar form to companion matrices, but with arbitrary diagonal.

This work is motivated by some earlier work of Šmigoc, who showed that diagonal elements are of importance to constructive methods in the NIEP.

Keywords

Nonnegative Matrices, Nonnegative Inverse Eigenvalue Problem, Diagonal Elements, Companion Matrix