Hurwitz and Hurwitz-type matrices of two-way infinite series

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Abstract

A function is stable or Hurwitz-stable if all its zeros lie in the left half of the complex plane. The classical approach to the Hurwitz stability (dating back to Hermite and Biehler) exploits a deep relation between stable functions and mappings of the upper half of the complex plane into itself (i.e. $\mathcal{R}$-functions). Hurwitz introduced a connection between minors of the Hurwitz matrix and the Hankel matrix built from coefficients of the corresponding $\mathcal{R}$-function (moments), which resulted in the famous Hurwitz criterion.

More recent studies [1, 6] highlighted another property related to the Hurwitz stability: the total nonnegativity of corresponding Hurwitz matrices, that is nonnegativity of all their minors. The paper [2] extends the criterion [5] to a complete description of power series (singly infinite or finite) with totally nonnegative Hurwitz matrices. During my talk, I am going to extend this result further to two-way (i.e. doubly) infinite power series. The corresponding general case of the necessary conditions [4, Theorem 4] for total nonnegativity of generalized Hurwitz matrices follows as an application.

The study is prompted by the criterion [3], because each Hurwitz matrix is built from two Toeplitz matrices. The essential connection to Hankel matrices breaks here (no correspondent Stieltjes continued fraction), and thus the doubly infinite case requires an approach distinct from the singly infinite case.

Keywords

Total positivity, Pólya frequency sequence, Hurwitz matrix, Generalized Hurwitz matrix, Doubly infinite series.

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