MAT TRIAD 2017

Book of Abstracts

September 25 – 29, 2017
Będlewo, Poland

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Contents

Part I. Introduction

Part II. Program

Part III. Lectures

Part IV. Invited Speakers

Part V. Special Sessions

Part VI. Contributed Talks

Part VII. Posters

Part VIII. Jaroslav Zemánek in memoriam

Part I

Introduction

The International Conference on Matrix Analysis and its Applications, MAT TRIAD 2017, which is the 7th from the MatTriad series, will be held on September 25-29, 2017 in Będlewo (neighborhood of Poznań, Poland) at the Mathematical Research and Conference Center of the Polish Academy of Sciences.

MatTriad provides an opportunity to bring together researchers sharing an interest in a variety of aspects of matrix analysis and its applications in other areas of science.

Matrix theory is used in almost all other parts of mathematics and all areas to which mathematics is applied, and, in return, other parts of mathematics can be very useful in proving things about matrices, sometimes things that are very difficult or impossible to prove using conventional matrix theoretic methods. For example there are many topics in which graph theory is useful in matrix theory, and, on the other hand, matrix theory is an indispensable tool in graph theory. Yet another connection is related to polynomials (possibly in several variables) that take on positive values only. This involves many matrix problems both ways. Many advances have been made recently, both theoretical and practical (connections with semidefinite programming), with a lot of discussion between matrix people and others mathematicians.

Researchers and graduate students interested in recent developments in matrix and operator theory and computation, spectral problems, applications of linear algebra in statistics, statistical models, matrices and graphs as well as combinatorial matrix theory are particularly encouraged to attend the conference. The format of the meeting will join together plenary talks and sessions with contributed talks. The list of invited speakers will be opened by the winners of the Young Scientists Awards of MAT-TRIAD 2015 (YSA 2015) promoted by the conference. We are also planning two short courses delivered by experienced lecturers for graduate students as well as others conference participants.

The work of young scientists continues to have a special position in the MAT-TRIAD 2017. The best talk / best poster of graduate students or scientists with a recently completed Ph.D. will be awarded. Prize-winning works will be widely publicized and promoted by the conference.

Covered topics:

- Spectral properties of matrices
- Matrix computations and numerical linear algebra
- Matrix inequalities
- Matrices and graphs
- Matrix polynomials
- Positive and nonnegative matrices
- Linear algebra in statistics
- Stable matrices

Call for Papers

We are pleased to announce a Special Issue of *ELA - Electronic Journal of Linear Algebra* devoted to MatTriad'2017. It will include selected papers strongly correlated to the talks presented during the conference.

Guest Editors:

Oskar M. Baksalary (Poland)
Natália Bebiano (Portugal)
Heike Faßbender (Germany)
Simo Puntanen (Finland)

All papers submitted must meet the publication standards of ELA (see: `http://repository.uwyo.edu/ela`) and will be subject to normal refereeing procedure. Authors should submit a paper through the ELA portal or to one of the special editors and clearly indicate that the paper is to be considered for this special volume. The deadline for submission of papers is **the end of December, 2017**.

Organizers:

- Banach Center, Institute of Mathematics, Polish Academy of Sciences, Poland
- Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Poznań, Poland
- Institute of Socio-Economic Geography and Spatial Management, Adam Mickiewicz University, Poznań, Poland
- Department of Mathematical and Statistical Methods, Poznań University of Life Sciences, Poland

Committees

The Scientific Committee for this Conference comprises

- Natália Bebiano (Portugal)
- Ljiljana Cvetković (Serbia)
- Heike Faßbender (Germany)
- Simo Puntanen (Finland)
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- Augustyn Markiewicz – Chair (Poland)
- Aneta Sawikowska (Poland)
- Dominika Wojtera-Tyrakowska (Poland)

Invited lectures:

- Lynn R. LaMotte (USA):
  Deconstructing type III
- Fuzhen Zhang (USA):
  Tensors and some combinatorial properties of tensors

Invited speakers:

- Narayanaswamy Balakrishnan (Canada):
  Multivariate stochastic comparisons in actuarial science and applications
- Andreas Frommer (Germany):
  Computing $f(A)b$, the action of a matrix function on a vector
- Volker Mehrmann (Germany):
  The distance to instability and singularity for structured matrix pencils
- K. Manjunatha Prasad (India):
  Introduction to rank-function and its applications
- Mirloslav Rozložník (Czech Republic):
  Numerics of the Gram-Schmidt process and its relation to the SR decomposition

Winners of YSA 2015:

- Jolanta Pielaszkiewicz (Sweden):
  On negative spectral moments using asymptotic freeness of matrices
- Ernest Šanca (Serbia):
  Robust dynamics of real systems based on the pseudospectra

Mat-Triad 2015 - group photo

Part II

Program

Monday, September 25, 2017

8:50 – 9:00 *Opening*

Invited Session 1

9:00 – 9:45 Volker Mehrmann: *The distance to instability and singularity for structured matrix pencils*

9:45 – 10:00 Break

Lecture 1, part I

10:00 – 11:15 Fuzhen Zhang: *Tensors and some combinatorial properties of tensors*

11:15 – 11:40 Coffee Break

Special Session 1, part I – Interval Matrices

11:40 – 12:10 Milan Hladík: *AE regularity of interval matrices*
12:10 – 12:35 Jaroslav Horáček: *Application of interval linear algebra in data estimation*
12:35 – 13:00 Jan Bok: *Interval pseudoinverse matrices*

13:00 – Lunch

Invited Session 2

15:00 – 15:45 K. Manjunatha Prasad: *Introduction to rank-function and its applications*

15:45 – 16:10 Coffee Break

Special Session 2, part I – Matrix Methods in Linear Models

16:10 – 16:40 Daniel Klein: *Estimation of parameters under the multilevel multivariate models*
16:40 – 17:10 Dietrich von Rosen: *Applications of the Vandermonde matrix in statistics*
17:10 – 17:40 Charles Rohde: *Robustness in the multivariate Gaussian distribution*

17:40 – 17:50 Break

Special Session 3, part I – Total Positivity

17:50 – 18:25 Jürgen Garloff: *Recent applications of the Cauchon algorithm to the totally nonnegative matrices*
18:25 – 18:50 Mikhail Tyaglov: *Root location of polynomials with totally nonnegative Hurwitz matrix*
18:50 – 19:15 Alexander Dyachenko: *Hurwitz and Hurwitz-type matrices of two-way infinite series*

19:30 – Barbecue

Tuesday, September 26, 2017

Invited Session 3

9:00 – 9:45 Miroslav Rozložník: *Numerics of the Gram-Schmidt process and its relation to the SR decomposition*

9:45 – 10:00 Break

Lecture 1, part II

10:00 – 11:15 Fuzhen Zhang: *Tensors and some combinatorial properties of tensors*

11:15 – 11:40 Coffee Break

Special Session 2, part II – Matrix Methods in Linear Models

11:40 – 12:10 Julia Volaufova: *Fixed effects estimation in two-variance components models*
12:10 – 12:40 Roman Zmyślony: *Application of Jordan algebra and its inference in linear models*
12:40 – 13:00 Célia Nunes: *Normal approximations for vec, trace and determinant of noncentral Wishart matrices*

13:00 – Lunch

15:00 – 15:45 Poster session

Péter Berkicks: *On parallel sum of matrices*
Célia Nunes: *Models for stochastic symmetric matrices*

15:45 – 16:10 Coffee Break

Contributed Session 1

16:10 – 16:40 Süleyman Solak: *On circulant matrices with Ducci sequences and Fibonacci numbers*
16:40 – 17:00 Osman Kan: *Commutators and matrix functions*
17:00 – 17:20 Mandeepa Saha: *Generalized Jacobi and Gauss-Seidel method for solving non-square linear systems*
17:20 – 17:40 Ryo Tabata: *Immanant inequalities on correlation matrices and Littlewood-Richardson's correspondence*

17:40 – 17:50 Break

Special Session – Jaroslav Zemánek in memoriam

17:50 – Andrzej Sołtysiak, Iwona Wróbel, Krystyna Ziętak, ...

19:30 – Dinner

20:30 – Concert

Wednesday, September 27, 2017

Invited Session 4

9:00 – 9:45 Andreas Frommer: *Computing $f(A)b$, the action of a matrix function on a vector*

9:45 – 10:00 Break

Lecture 2, part I

10:00 – 11:15 Lynn R. LaMotte: *Deconstructing type III*

11:15 – 11:40 Coffee Break

Special Session 3, part II – Total Positivity

11:40 – 12:05 Àlvaro Barreras: *Extending accurate computations for totally positive matrices*
12:05 – 12:30 Akiko Fukuda: *Integrable eigenvalue algorithms for totally nonnegative matrices*
12:30 – 12:55 Kanae Akaiwa: *Solving inverse eigenvalue problems for totally nonnegative matrices with finite steps*

13:00 – Lunch

14:00 – Excursion

Thursday, September 28, 2017

Invited Session 5

9:00 – 9:45 Jolanta Pielaszkiewicz: *On negative spectral moments using asymptotic freeness of matrices*

9:45 – 10:00 Break

Lecture 2, part II

10:00 – 11:15 Lynn R. LaMotte: *Deconstructing type III*

11:15 – 11:40 Coffee Break

Special Session 1, part II – Interval Matrices

11:40 – 12:00 Michal Černý: *A Branch-and-Bound scheme for the range of rank-deficient quadratic forms with interval-valued variables*
12:00 – 12:20 David Hartman: *Tightening bounds on the radius of nonsingularity*
12:20 – 12:40 Josef Matějka: *Determinants of interval matrices*
12:40 – 13:00 Iwona Skalna: *A new method for determining the radius of regularity of parametric interval matrices*

13:00 – Lunch

Invited Session 6

15:00 – 15:45 Ernest Šanca: *Robust dynamics of real systems based on the pseudospectra*

15:45 – 16:10 Coffee Break

Special Session 2, part III – Matrix Methods in Linear Models

16:10 – 16:40 Viktor Witkovský: *Tools for numerical inversion of the characteristic functions and their applications*
16:40 – 17:00 Arkadiusz Kozioł: *Best unbiased estimates for parameters of three-level multivariate data with doubly exchangeable covariance structure*
17:00 – 17:20 Monika Mokrzycka: *Kronecker product approximation via entropy loss function*
17:20 – 17:40 Adam Mieldzioc: *On projection of a positive definite matrix on a cone of non-negative definite Toeplitz matrices*

17:40 – 17:50 Break

Contributed Session 3

17:50 – 18:10 Pieter-Jan Pauwelyn: *Neglecting non-diagonalizable matrices in social sciences*
18:10 – 18:30 Terrence Teh: *Sums of H-unitary matrices*
18:30 – 18:50 Mika Mattila: *Studying the inertias of LCM matrices*
18:50 – 19:10 Roksana Słowik: *Infinite matrices and the Jordan form*

20:00 – Conference Dinner

Friday, September 29, 2017

Invited Session 7

9:00 – 9:45 Narayanaswamy Balakrishnan: *Matrix orders and stochastic orderings in actuarial science*

9:45 – 10:00 Break

Contributed Session 3

10:00 – 10:30 Natalia Bebiano: *Density matrices arising from incomplete measurements*
10:30 – 11:00 Damjana Kokol Bukovšek: *Linear spaces of symmetric nilpotent matrices*
11:00 – 11:20 Blaž Mojškerc: *Jordan triple product homomorphisms on triangular matrices to and from dimension one*

11:20 – 11:40 Coffee Break

Contributed Session 4

11:40 – 12:10 Susana Furtado: *Bruhat order for symmetric (0,1)-matrices*
12:10 – 12:40 Krystyna Ziętak: *Sub-Stiefel Procrustes problem*
12:40 – 13:00 Iwona Wróbel: *Both residual errors accurate algorithm for inverting general tridiagonal matrices*

13:00 – Lunch

Special Session 2, part IV – Matrix Methods in Linear Models

15:00 – 15:30 João T. Mexia: *Confidence regions and tests for normal models with orthogonal block structure: pivot variables*
15:30 – 16:00 Ivan Žežula: *Problems of inference in a special multivariate linear model*

16:00 – 16:30 Coffee Break

Special Session 2, part V – Matrix Methods in Linear Models

16:30 – 17:00 Simo Puntanen: *More about linear sufficiency in the linear mixed model*
17:00 – 17:20 Mariusz Grządziel: *On nonnegative minimum biased quadratic estimation in the linear regression models*

17:20 – 17:30 Break

Special Session 2, part VI – Matrix Methods in Linear Models

17:30 – 18:00 Katarzyna Filipiak: *Properties of partial trace and block trace operators of partitioned matrices*
18:00 – 18:20 Richard Ellard: *Diagonal elements in the nonnegative inverse eigenvalue problem*
18:20 – 18:40 Jan Hauke: *Algebraic properties of some contingency tables*
18:40 – *Closing*

19:00 – Dinner

Part III

Lectures

Deconstructing type III

Lynn R. LaMotte

LSU Health - New Orleans U.S.A.

R. A. Fisher expounded analysis of variance (ANOVA) for settings in which responses are observed under experimental conditions described by combinations of levels of one or more factors. For two factors, ANOVA partitions differences among cell means into three groups, "variation between classes of type A and between classes of type B" and "interaction of causes" (Fisher 1938, p. 240), commonly named A and B main effects and AB interaction effects. This scheme extends readily, resolving differences for f factors into $2^f - 1$ effects. For three factors, for example, the effects are the three main effects, three two-factor interaction effects, and the three-factor interaction effect.

ANOVA quickly became the main statistical methodology in diverse disciplines. It provides concepts and terminology that have become a common language of applied statistics.

In balanced models, with equal numbers of observed responses over all factor-level combinations (FLCs or cells), sums of squares (SSs) for statistics to test the effects have simple formulations, and computing them is straightforward. Their distributional properties (assuming normally-distributed responses) are apparent.

The situation is unsettled for unbalanced models and settings in which there are no observations in some cells. ANOVA SSs do not provide appropriate test statistics. While there is no theoretical impediment to mimicking the balanced-model partition, the results are less (often un-) informative because some or all subspaces of effects are not estimable. No statistical computing package, as far as I know, takes this approach.

Most packages use Type III SSs to produce an ANOVA-like partition of effects. Type III was introduced by SAS in the 1970s: see Goodnight (1976) and SAS (1978). It is defined by an algorithm; the process begins and ends with a set of rules for formulating the Type III estimable functions of an effect. No hypothesis is formulated. The SS for an effect is the squared norm of the orthogonal projection of the vector of observed responses onto the space of Type III estimable functions.

The lack of a compact mathematical definition of Type III SSs gives them a mystical, black-box aura. A system of beliefs has evolved. For example, it is widely asserted that Type III SSs are the same as ANOVA SSs in balanced models, and that they test ANOVA-effect hypotheses in unbalanced models

if there are no empty cells. Proofs of these assertions do not exist. Apparently what is thought to be known about Type III is based only on experience and observation.
My objective in this lecture is to provide an explicit formulation of Type III estimable functions and SSs and to establish their properties. It is shown that what is believed to be true, is true, when all of an effect is estimable, but not otherwise.

Keywords

ANOVA, Factor effects.

References

[1] Fisher, R.A. (1938). *Statistical Methods for Research Workers, 7th Edition.* Oliver and Boyd, London.
[2] Goodnight, J.H. (1976). The general linear models procedure. *Proceedings of the First International SAS User's Group.* SAS Institute Inc., Cary, NC.
[3] SAS Institute Inc. (1978). SAS Technical Report R-101, *Tests of hypotheses in fixed-effects linear models.* SAS Institute Inc., Cary, NC.

Tensors and some combinatorial properties of tensors

Fuzhen Zhang

Nova Southeastern University, Fort Lauderdale, Florida, USA

Abstract: We begin with the definition of a tensor (in algebra) and then focus on the tensors by which we mean multi-dimensional arrays (or hypermatrices) of real numbers. A square matrix is doubly stochastic if its entries are all nonnegative and each row and column sum is 1. A celebrated result known as Birkhoff's theorem about doubly stochastic matrices states that an $n \times n$ matrix is doubly stochastic if and only if it is a convex combination of some $n \times n$ permutation matrices (a.k.a Birkhoff polytope). The Birkhoff polytope of $n \times n$ stochastic matrices in $\mathbb{R}^{n^2}$ is of dimension $(n-1)^2$ with n^2 facets and $n!$ vertices.
We consider the generalization of the Birkhoff's theorem in higher dimensions. An $n \times n \times n$ stochastic tensor is a nonnegative array (hypermatrix) in which every sum over one index is 1. A permutation tensor can be identified with a Latin square (vice versa). We study the polytope of all these tensors, the convex set of all tensors with some positive diagonals, and the polytope generated by the permutation tensors. We present lower and upper bounds for the number of vertices of the polytopes, and discuss further questions on the topic.
Determinant and permanent are basic and important functions of $n \times n$ matrices. We attempt to define these for tensors. More generally, we will consider defining the generalized matrix functions for tensors.

Keywords

Birkhoff polytope, Doubly stochastic matrix, Extreme point, Hypermatrix, Polytope, Stochastic semi-magic cube, Stochastic tensor, Tensor, Vertex.

References

[1] Juflkat, W.B. and H.J. Ryser (1968). Extremal configurations and decomposition Theorems. I. *J. Algebra 8*, 194–222.
[2] Li, Z., F. Zhang, and X.-D. Zhang. On the number of vertices of the stochastic tensor polytope. *Linear Multilinear Algebra.* To appear.
[3] Ziegler, G.M. (1995). *Lectures on Polytopes.* Springer, New York.

Part IV

Invited Speakers

Multivariate stochastic comparisons in actuarial science and applications

Narayanaswamy Balakrishnan

McMaster University, Hamilton, Canada

In this talk, I will first introduce the notions of univariate stochastic orderings, a technique by which two random variables can be compared. I will then describe some multivariate orderings. I will then consider the total claim amount from two portfolios in an actuarial setup and apply these univariate and multivariate orderings to present some results. Finally, I will also present a multivariate stochastic ordering result for the whole set of order statistics drawn from a distribution. I will present some illustrative examples through out to explain the results obtained.

Computing $f(A)b$, the action of a matrix function on a vector

Andreas Frommer

Bergische Universität Wuppertal, Germany

Let A be a square matrix and f a function that is sufficiently smooth on the spectrum of A. Then the matrix function $f(A)$ is defined as $p(A)$, where p is the polynomial that interpolates f on the spectrum of A in the Hermite sense. Practically, it is impossible to compute $fA)$ when A is big and sparse, while it is still possible to compute $f(A)b$, b a vector.
In our talk we will address Krylov subspace techniques for computing $f(A)b$. The emphasis will be on stable restart procedures, which are mandatory even in the case that A is Hermitian, and on convergence theory. We will dedicate a large part of the talk to extensions to block methods, where one is interested in $f(A)B$, the columns of B representing several vectors b. In this contect we develop a fairly general theory for a class of block Krylov methods which comprises several different block methods considered in the literature before. We will again focus on restart procedures and convergence analysis.
This talk is based on joint work with Stefan Güttel, Kathryn Lund, Marcel Schweitzer and Daniel B. Szyld.

Keywords

Matrix functions, Krylov subspaces, Stability, Block methods, Convergence analysis, Stieltjes functions.

The distance to instability and singularity for structured matrix pencils

Volker Mehrmann

Technishe Universität Berlin, Germany

The stability analysis of dynamical systems leads to the eigenstructure analysis of matrix pencils $\lambda E - A$. The associated system is asymptotically stable if the pencil is regular, all finite eigenvalues are in the left half plane and the infinite eigenvalues are semisimple. There are several challenging open problems that will be discussed. The first is the distance to instability, i.e. the smallest perturbation to E and A that puts an eigenvalue on the imaginary axis, or makes the pencil singular. For the first question there are well-known methods but the second problem is still open, although progress has been recently made, [5,2]. When the problem is structured such as in port-Hamiltonian systems the distance to instability is much larger than for the unstructured case [3,4]. This opens a lot of opportunities to exploit the structure to the advantage of robustness of a system under perturbation. We also discuss the converse problem of finding the distance to the boundary of the stable pencils for a given unstable pencil [1].
This presents joint work in different publications with N. Gillis, N. Guglielmi, C. Lubich, C. Mehl, P. Sharma, and M. Wojtylak.

Keywords

Distance to instability, Distance to the nearest singular pencil, Nearest stable pencil, Structured distances, Port-Hamiltonian system.

References

[1] Gillis, N., V. Mehrmann, and P. Sharma (2017). Computing nearest stable matrix pairs. Preprint 04-2017, Inst. Mathematics, TU Berlin. `https://arxiv.org/pdf/1704.03184.pdf`. Submitted for publication.

[2] Guglielmi, N., C. Lubich, and V. Mehrmann (2016). On the nearest singular matrix pencil. Preprint 12-2016, Inst. Mathematics, TU Berlin. *SIAM J. Matrix Anal. Appl.* To appear.

[3] Mehl, C., V. Mehrmann, and P. Sharma (2016). Stability radii for linear Hamiltonian systems with dissipation under structure-preserving perturbations. *SIAM J. Matrix Anal. Appl. 37*, 1625–1654.

[4] Mehl, C., V. Mehrmann, and P. Sharma (2017). Stability radii for real linear Hamiltonian systems with perturbed dissipation. Preprint 1110, Research Center Matheon, Institute of Mathematics, TU Berlin. *BIT Numerical Mathematics.* To Appear. Available at: `doi:10.1007/s10543-017-0654-0`.

[5] Mehl, C., V. Mehrmann, and M. Wojtylak (2015). On the distance to singularity via low rank perturbations. *Oper. Matrices 9*, 733–772.

On negative spectral moments using asymptotic freeness of matrices

Jolanta Pielaszkiewicz

Linnaeus University, Växjö, Sweden

There is a number of matrices that are proven to be asymptotically free. For such matrices additive properties of R-transform (free cumulant generation function) can be used to derive number of results related to negative spectral moments. Some particular matrix polynomials will be considered with the special interest in polynomials in Wishart matrices. Relation between the spectral moment generating function of matrix and its inverse and closed form expression for R-transform of Inverse Wishart matrix will be given. The talk will be illustrated with comparison of theoretical and simulations results.

Keywords

Negative moments, Wishart matrix, Trace, Freeness, R-transform.

References

[1] Cressie, N., A.S. Davis, J.L. Folks, and G.E. Policello II (1981). The moment-generating function and negative integer moments. *Amer. Statist. 35(3)*, 148–150.

[2] Pielaszkiewicz, J. (2015). *Contributions to High-Dimensional Analysis under Kolmogorov Condition.* Linköpings Studies in Science and Technology. Dissertation No. 1724.

[3] Pielaszkiewicz, J., D. von Rosen, and M. Singull (2017). On $E\left[\prod_{i=0}^{k} Tr\{W^{m_i}\}\right]$, where $W \sim \mathcal{W}_p(I, n)$. *Comm. Statist. Theory Methods 46*, 2990–3005.

[4] von Rosen, D. (1997). On moments of the inverted Wishart distribution. *Statistics 30(3)*, 259–278.

Introduction to rank-function and its applications

Nayan Bhat, K. Manjunatha Prasad, and Nupur Nandini

Manipal University, India

The notion of 'rank of a matrix' as defined by 'the dimension of subspace generated by columns of that matrix' over any field has limitation to be considered for a matrix over any other algebraic structure. The 'determinantal rank' defined by the size of largest submatrix having nonzero determinant, which is in fact equivalent to the column rank for any matrix over a field, was considered to be an alternative for the class of matrices over a commutative ring. Even this determinantal rank or the McCoy rank are not so efficient in describing several properties of matrices like in the case of solvability of linear system. In the present talk, we discuss the introduction of 'rank function' associated with the matrix as defined in [4] and its characteristics. Also, we present rank condition for the existence of Drazin inverse for a square matrix over a commutative ring.

Keywords

Rank, Determinantal rank, Rank function, Generalized inverse, Drazin inverse.

References

[1] Ben-Israel, A. and T.N.E. Greville (1974). *Generalized Inverses: Theory and Applications*. Wiley-Interscience, New York.

[2] Camion, P., L.S. Levy, and H.B. Mann (1971). Linear equation over a commutative ring. *J. Algebra 18*, 432–446.

[3] Ching, W.-S. (1977). Linear equations over commutative rings. *Linear Algebra Appl. 18*, 25–266.

[4] Manjunatha Prasad, K. (1997). Solvability of linear equations and rank function. *Comm. Algebra 25(1)*, 297–302.

[5] Rao, C.R. and S.K. Mitra (1971). *Generalized Inverse of Matrices and Its Applications*. Wiley, New York.

Numerics of the Gram-Schmidt process and its relation to the SR decomposition

Miroslav Rozložník

Czech Academy of Sciences, Prague, Czech Republic

In this contribution we first consider the most important schemes used for orthogonalization with respect to the standard and non-standard inner product and briefly review the main results on their behavior in finite recision arithmetic. We treat separately the particular case of the standard inner product and show that similar results hold also for the case when the inner product is induced by a positive diagonal matrix. We will show that in the general case of non-standard inner product the conditioning of computed factors depends not only on the conditioning of initial vectors but it depends also on the condition number of the matrix that induces the non-standard inner product. We also study the orthogonalization schemes for computing vectors that are mutually orthogonal with respect to the bilinear form induced by a symmetric nonsingular but indefinite matrix. Under assumption on strong nonsingularity of this matrix we develop bounds for the extremal singular values of the triangular factor that comes from is symmetric indefinite factorization. It appears that they depend on the the extremal singular values of the matrix and of only those principal submatrices where there is a change of sign in the associated subminors. Using these results we analyze two types of schemes used for orthogonalization and we give the worst-case bounds for quantities computed in finite precision arithmetic. In particular, we consider Cholesky QR implementation based on the Cholesky-like factorization and the Gram-Schmidt process with respect to this bilinear form. We consider also their versions with reorthogonalization and with one step of iterative refinement.
Finally we discuss the extension of this theory to the case of skew-symmetric bilinear forms used in the context of various structure-preserving transformations. We analyze the freedom of choice in the symplectic and the triangular factors and review several existing suggestions on how to choose the free parameters in the SR decomposition.

Keywords

Orthogonalization, Gram-Schmidt process, Indefinite inner product, Skew-symmetric bilinear form, SR decomposition.

Acknowledgement

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References

[1] Faßbender, H. and M. Rozložník (2016). On the conditioning of factors in the SR decomposition. *Linear Algebra Appl. 505*, 224–244.

[2] Rozložník, M., J. Kopal, M. Tuma, and A. Smoktunowicz (2012). Numerical stability of orthogonalization methods with a non-standard inner product. *BIT Numerical Mathematics 52*, 1035–1058.

[3] Rozložník, M., F. Okulicka-Dłużewska, and A. Smoktunowicz (2015). Cholesky-like factorization of symmetric indefinite matrices and orthogonalization with respect to bilinear forms. *SIAM J. Matrix Anal. Appl. 36(2)*, 727–751.

Robust dynamics of real systems based on the pseudospectra

Ernest Šanca

University of Novi Sad, Serbia

The traditional approach to the stability analysis of the equilibria of real dynamical systems is based on Lyapunov stability, which consists of determining the position of the eigenvalues of the Jacobian in the complex plane. Such dynamical properties are primarily asymptotic in nature, therefore requiring usually long time scales for their realization. Additionally, potential transient behavior, which may violate the initial integrity of the system itself, when it is generating a functional response to the change it is facing, as well as the potential structural changes of the system itself, cannot be analyzed based on the expected asymptotic behavior only. The unity of rich mathematical theory, powerful theory of matrices and theory of dynamical systems offers knowledge regarding the existence of special matrix structures that are prone to high sensitivity of spectral properties, such as stability, in the presence of small perturbations. Moreover, this affectability to small perturbations in particular, is known to correspond to the effects of transient instability of otherwise asymptotically stable dynamical systems. Powerful mathematical tool that has been especially designed so as to provide better understanding of the aforementioned phenomena and adequate tool for its advanced analysis is known under the name pseudospectra. Stability indicators developed so far, exploiting the spectral properties of a matrix or the concept of GDD matrices, are simply lacking power and display drawbacks, thus providing motivation for the introduction of advancements as far as the methodology of the dynamical stability description is concerned, with the transient behavior under the functional changes in mind in the first place, highlighting the essence of pseudospectra.

This lecture aims to present concepts essentially familiar to Lyapunov stability of dynamical systems, which on one hand possess the necessary flexibility of applications in various fields of science, and the desired potential to describe complex stability aspects of real systems on the other hand. Ultimately, efficient numerical methods in determining these generalized aspects of stability, based on empirical data, so as to enable practical applications of novel concepts, shall be presented. Finally, the desired possibility of implementation and utilization of developed concepts in the multidisciplinary ambiance including physics, ecology, medicine, chemistry, engineering, economy and many more, shall be discussed.

Part V

Special Sessions

Special Session on Total Positivity

Jürgen Garloff

University of Applied Sciences, Konstanz, Germany
University of Konstanz, Germany

The concept of total positivity is rooted in classical mathematics where it can be traced back to works of Schoenberg on variation diminishing properties and of Gantmacher and Krein on small oscillations of mechanical systems. Since then the class of totally positive matrices and operators proved to be relevant in such a wide range of applications that over the years many distinct approaches to total positivity, amenable to a particular notion, have arisen and advocated by many prominent mathematicians. This area is, however, not just a historically significant subject in mathematics, but the one that continues to produce important advances and spawn worth-wile applications. This is reflected by the topics which will be covered by the speakers of the Special Session, viz. the study of classes of matrices related to total positivity, accurate computations based on bidiagonalization, inverse eigenvalue problems, log-concavity, and the location of the roots of polynomials.

Keywords

Total positivity, Bidiagonalization, Inverse eigenvalue problems, Location of the roots of polynomials, log-concavity.

Invited speakers:

K. Akaiwa, A. Barreras, A. Dyachenko, A. Fukuda, M. Tyaglov.

Special Session on Interval Matrices

Milan Hladík

Charles University, Czech Republic

An interval matrix is defined as a set of matrices lying entrywise between two given matrices. An interval matrix is a fundamental notion in interval computation, which is focused on rigorous computation with real or interval data. The key property of interval computation is the "enclosing property", guaranteeing that all possible realizations of interval data and all roundoff errors are taken into account. Due to this property, interval computation is an important tool in verification in numerical analysis, global optimization, constraint programming and many other areas. Therefore, handling interval matrices is a very frequent problem in interval computation since one often needs to verify some matrix property (nonsingularity etc.), approximate its eigenvalues, or solve an interval linear system of equations.
This special session will be devoted to investigation of various properties of interval matrices, including theoretical characterization, developing efficient algorithms, classification in the computational complexity sense, as well as solving interval linear systems of equations and the related problems.

Invited speakers:

J. Bok, M. Černý, D. Hartman, J. Horáček, J. Matějka.

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`http://www.nsc.ru/interval/Library/InteBooks/!handbook.pdf`

Special Session on Matrix Methods in Linear Models

Daniel Klein

P. J. Šafárik University, Košice, Slovakia

Linear models are everywhere in data analysis. In spite of the availability of highly innovative tools in statistics, the linear models are still widely studied by statisticians. Even the most effective multivariate models appear to be the linear ones. To describe these models it is most efficient with matrix algebra, it is the language of modern analysis. Also the study of various concepts would be tedious without matrix algebra.
This special session will be devoted to estimation and testing problems in multivariate and mixed linear models, where the application of matrix algebra and tensor operators plays a crucial role. The results on determination of estimators of unknown parameters, on characterization of their properties or comparison of different estimators, as well as procedures of testing hypotheses devoted to structured mean or variance-covariance matrix are mostly welcome to this session.

Invited speakers:

K. Filipiak, J. T. Mexia, D. von Rosen, J. Volaufova, R. Zmyślony.

Part VI

Contributed Talks

Solving inverse eigenvalue problems for totally nonnegative matrices with finite steps

Kanae Akaiwa

Kyoto Sangyo University, Japan

Inverse eigenvalue problems include a problem of constructing structured matrices with prescribed eigenvalues. Construction of totally nonnegative (TN) matrices, whose minors are all nonnegative, with prescribed eigenvalues is an important topic of inverse eigenvalue problems. In this talk, it is clarified that an inverse eigenvalue problem for TN matrices is closely related with some integrable systems, where integrable systems mean dynamical systems which have exact solutions. In particular, it is shown that TN matrices with prescribed eigenvalues can be constructed in finite steps with the help of discrete integrable systems such as the discrete Toda equation.

Extending accurate computations for totally positive matrices

Àlvaro Barreras[1] and Juan M. Peña[2]

Universidad de Zaragoza, Spain

It is known that for totally positive matrices, their bidiagonal decomposition is an adequate parametrization in order to carry out accurate computations. Given these parameters, it is possible to perform subtraction- free algorithms to compute the inverse matrix, eigenvalues and singular values (see [7]). In this talk we recover the method presented in [2] to extend these algorithms to ε-SBD matrices, a class of matrices that contains not only totally positive matrices but also their inverses (see also [1]). We also extend the results of Koev (cf. [7]) to new classes of matrices, preserving the accuracy independently of its conditioning (see [3]). Joint work with Juan Manuel Peña.

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Density matrices arising from incomplete measurements

Natalia Bebiano

University of Coimbra, Portugal

In this paper, the following problem is considered: given two Hermitian matrices H and K and two real numbers x and y, determine a positive semidefinite matrix ρ such that the von Neumann entropy $-\mathrm{Tr}_\rho \log \rho$ is maximum, subject to the condition that $\mathrm{Tr}_\rho H = x$ and $\mathrm{Tr}_\rho K = y$. This question arises in information theory and in statistical mechanics in connection with the maximum-entropy inference principle. To answer it, we use this principle and numerical range methods.

Interval pseudoinverse matrices

Jan Bok and **Milan Hladík**

Charles University, Prague, Czech Republic

One of the main problems in interval linear algebra is to decide for some given interval matrix if it is regular. An interval matrix is regular if all its selections are regular. In classic linear algebra, a natural approach in the case that a given matrix is singular is to find a pseudoinverse matrix which is in some sense very close to being the inverse matrix. The most widely used notion is the Moore-Penrose pseudoinverse matrix. This type of matrix can be generalized to interval matrices as well.

We will talk about our recent results on interval pseudoinverse matrices. We will present both theoretical and experimental results regarding different approaches to interval pseudoinverse computation and its tightness. We will compare our results with the work of Saraev [1], to our knowledge the only existing paper dealing with interval pseudoinverse so far.

Keywords

Interval analysis, Interval matrix, Pseudoinverse.

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A Branch-and-Bound scheme for the range of rank-deficient quadratic forms with interval-valued variables

Michal Černý[1], Miroslav Rada[1], and Milan Hladík[1,2]

[1] University of Economics, Prague, Czech Republic
[2] Charles University, Prague, Czech Republic

Given a quadratic form $f(x) = x^T Qx$ and bounds $\underline{x} \le x \le \overline{x}$ for its variables, we address the problem of computing the range $\underline{f} = \min_{\underline{x} \le x \le \overline{x}} f(x)$ and $\overline{f} = \max_{\underline{x} \le x \le \overline{x}} f(x)$. First we address the case when Q is positive semidefinite. Then the lower bound $\underline{f}$ can be computed efficiently via CQP, while computation of the upper bound $\overline{f}$ is NP-hard. We focus on the case when Q is rank-deficient. We reformulate the computation of $\overline{f}$ as a problem of enumeration of vertices of a zonotope in d-dimensional space [4], where $d = \text{rank}(Q)$. Instead of constructing the enumeration of vertices in full (as in [1,5]), we design a B&B scheme. The branching step consists in a split of a zonotope into a pair of "smaller" zonotopes by removal of a generator. In the bound-part, we use Goffin's method [2] to approximate a zonotope by a pair of Löwner-John ellipsoids. Then, the lower and upper bound for f over an ellipsoid is computed by Ye's algorithm [6] for optimization of (arbitrary) quadratic forms over ellipsoids. We also discuss the impact of various strategies for the choice of (i) the active zonotope, (ii) the branching generator and (iii) the method for computation of lower bounds.

The general case, when Q need not be positive semidefinite (but is still rank-deficient), can be reduced to the problem of enumeration of all k-dimensional faces of a $2d$-dimensional zonotope, where $k = 0, 1, \ldots, 2d$. This can be done by zonotope enumeration algorithms [3]. We design a B&B strategy for this case, too. Here, the branching step consists in replacement of a zonotope by a pair of zonotopes with a shorter generator. The bounding step is similar to the psd case. This B&B scheme generates a potentially infinite branching tree with a branch converging to the maximizer/minimizer. Cutting the tree at a certain level allows us to compute an ε-approximate solution.

Keywords

Quadratic form, Interval data, Zonotope, Branch and Bound.

Acknowledgement

This work was partially supported by CSF 16-00408S.

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Hurwitz and Hurwitz-type matrices of two-way infinite series

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A function is stable or Hurwitz-stable if all its zeros lie in the left half of the complex plane. The classical approach to the Hurwitz stability (dating back to Hermite and Biehler) exploits a deep relation between stable functions and mappings of the upper half of the complex plane into itself (i.e. $\mathcal{R}$-functions). Hurwitz introduced a connection between minors of the Hurwitz matrix and the Hankel matrix built from coefficients of the corresponding $\mathcal{R}$-function (moments), which resulted in the famous Hurwitz criterion.

More recent studies [1,6] highlighted another property related to the Hurwitz stability: the total nonnegativity of corresponding Hurwitz matrices, that is nonnegativity of all their minors. The paper [2] extends the criterion [5] to a complete description of power series (singly infinite or finite) with totally nonnegative Hurwitz matrices. During my talk, I am going to extend this result further to two-way (i.e. doubly) infinite power series. The corresponding general case of the necessary conditions [4, Theorem 4] for total nonnegativity of generalized Hurwitz matrices follows as an application.

The study is prompted by the criterion [3], because each Hurwitz matrix is built from two Toeplitz matrices. The essential connection to Hankel matrices breaks here (no correspondent Stieltjes continued fraction), and thus the doubly infinite case requires an approach distinct from the singly infinite case.

Keywords

Total positivity, Pólya frequency sequence, Hurwitz matrix, Generalized Hurwitz matrix, Doubly infinite series.

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Diagonal elements in the nonnegative inverse eigenvalue problem

Richard Ellard and **Helena Šmigoc**

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We say that a list of complex numbers is *realisable* if it is the spectrum of some (entrywise) nonnegative matrix. The Nonnegative Inverse Eigenvalue Problem (NIEP) is the problem of characterising all realisable lists.
Although the NIEP remains unsolved, it has been solved in certain cases. In particular, the solution is known if the list contains at most three elements or if every entry in the list (apart from the Perron eigenvalue) has nonpositive real part. In these cases, if a realising matrix is known to exist, one may ask what the possible diagonal elements of said matrix are. For a given realisable spectrum, we show that a list of nonnegative numbers may arise as the diagonal elements of the realising matrix if and only if these numbers satisfy a remarkably simple inequality. The realising matrices employed are of a similar form to companion matrices, but with arbitrary diagonal.
This work is motivated by some earlier work of Šmigoc, who showed that diagonal elements are of importance to constructive methods in the NIEP.

Keywords

Nonnegative matrices, Nonnegative inverse eigenvalue problem, Diagonal elements, Companion matrix.

Properties of partial trace and block trace operators of partitioned matrices

Katarzyna Filipiak[1] **and Daniel Klein**[2]

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The aim of this paper is to give the properties of two linear operators defined on $pq \times mq$ partitioned matrix $\mathbf{A} = (\mathbf{A}_{ij})$ with $q \times q$ blocks $\mathbf{A}_{ij}$:

- partial trace operator, $\mathrm{PTr}_q \mathbf{A} = (\mathrm{tr}\mathbf{A}_{ij})$ (cf. [1]), and
- block trace operator for $m = p$, $\mathrm{BTr}_q \mathbf{A} = \sum_{i=1}^{p} \mathbf{A}_{ii}$.

The conditions for symmetry, nonnegativity, and positive-definiteness of the operators are given, as well as the relations between partial trace and block trace operators with standard trace, vectorizing and Kronecker product operators.
Both partial trace and block trace operators can be widely used in statistics, for example in the estimation of unknown parameters under the multi-level multivariate models or in the theory of experiments for determination of optimal designs under linear models.

Acknowledgement

This talk is partially supported by Statutory Activities No. 04/43/DSPB/0088.

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Integrable eigenvalue algorithms for totally nonnegative matrices

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There are interesting relationships between eigenvalue algorithms and integrable systems. Integrable systems are nonlinear differential or difference equations which can be solved exactly. Based on the integrable discrete hungry Toda molecule equation, we have designed an algorithm for computing eigenvalues of a class of totally nonnegative matrices [1]. This algorithm can be regarded as a generalization of the dqds algorithm. In this talk, we focus on the discrete two-dimensional Toda molecule (d2Toda) equation, which is a generalization of the discrete hungry Toda molecule equation. We show that the d2Toda equation can be applied to compute eigenvalues of a class of totally nonnegative (TN) matrices. Through discrete time evolution of the d2Toda equation, the d2Toda variables yield the eigenvalues of the TN matrix. The resulting algorithm can be regarded as an extension of the qd algorithm. We also show the relationship between another integrable system and TN matrices, and computation of the eigenvector of the TN matrices.

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Bruhat order for symmetric $(0,1)$-matrices

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Extending the Bruhat order for permutation matrices, in [2] a Bruhat order for the class of m-by-n $(0,1)$-matrices with prescribed row and column sum vectors was defined. Minimal matrices for this Bruhat order (a partial order) were studied in this paper and in the subsequent paper [1].
In this talk we present some results, obtained in [3], related with the description of the minimal matrices in the Bruhat order for the class of symmetric $(0,1)$-matrices with given row sum vector. We start by giving some properties of these minimal matrices. We also present minimal matrices in the Bruhat order for some particular such classes of symmetric $(0,1)$-matrices. Some connections with the term rank of a matrix will be established.

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Recent applications of the Cauchon algorithm to totally nonnegative matrices

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[2] University of Konstanz, Germany
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The Cauchon algorithm, see, e.g., [4], has been applied to totally nonnegative matrices in order to characterize these matrices [4] and their subclasses [1], to recognize totally nonnegative matrix cells [5], and to derive determinantal criteria for this class of matrices [1]. In this talk we report on some recent applications of this algorithm, e.g., to the study the invariance of total nonnegativity under element-wise perturbation and the subdirect sum of two totally nonnegative matrices [2], to the investigation of the interval property of sign regular matrices, and to the determination of the rank of an arbitrary matrix [3].

Keywords

Cauchon algorithm, Totally nonnegative matrix, Subdirect sum, Interval property, Rank.

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On nonnegative minimum biased quadratic estimation in the linear regression models

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The problem of nonnegative estimation of a parametric function $\gamma(\beta,\sigma) = \beta' H\beta + h\sigma^2$ in the linear regression model $\mathcal{M}\{y, X\beta, \sigma^2 I\}$, where H is a nonnegative definite matrix and h is a nonnegative scalar, attracted attention of many researchers. Gnot et al. [2] proposed an approach in which γ is estimated by a quadratic form $y'Ay$, where A is a nonnegative definite matrix that satisfies an appropriate optimality criterion associated with minimizing the bias of the estimator. Computing the matrix A, which in the general case is not given explicitly, may be challenging.
A comparison of various approaches for finding A (developed e.g. in [2,1,3]) will be presented. The efficiency of these approaches will be illustrated by numerical examples.

Keywords

Linear regression model, Nonnegative minimum biased estimators, Mean squared error.

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Tightening bounds on the radius of nonsingularity

David Hartman and **Milan Hladík**

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Evaluating the proximity of a given square matrix to the nearest singular one can be performed via adopting Chebyshev norm leading to so called radius of nonsingularity. Let A be a matrix of a form $\mathbb{R}^{n\times n}$ and Δ is non-negative matrix of the same type, the *radius of nonsingularity* [2,3] is defined by

$$d(A,\Delta) := \inf\{\varepsilon > 0;\ (\exists \text{ singular } B)(\forall i,j) :\ |a_{ij} - b_{ij}| \leq \varepsilon \Delta_{i,j}\}.$$

There also exists a simplified version of such radius where Δ is equal to "all ones matrix" E. Determining exact value of this radius even in its simplified version is known to be an NP-hard problem [3], which leads to various lower and upper bounds [4,5]. These bounds, however, are not very tight - one of the best classical bounds has the relative error $6n$. We describe a better one based on a randomized approximation method with expected error 0.7834 using a semidefinite relaxation [1] and discuss its possible extensions depending on various conditions given.

Keywords

Radius of non-singularity, Regularity, Interval matrix, Bounds.

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Algebraic properties of some contingency tables

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A chi-squared test as one of the most popular nonparametric independence tests can be performed using contingency tables, which summarize the relationship between several categorical variables. From the algebraic point of view, a contingency table as a two-way table is a matrix of non-negative integers showing cross-classiffication.
The idea of using such tables for the test was proposed by [2] and is based on comparing the "distance" of the empirical array of contingencies with its theoretical counterpart - expressing the full invariance. The properties of the contingency tables are still in the interest of researchers (see [1]). The aim of the paper is to analyse some algebraic properties of a square and symmetric contingency tables as matrices and their theoretically equalized counterparts.

Keywords

Contingency table, Chi squared test.

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AE regularity of interval matrices

Milan Hladík

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Consider a linear system of equations with interval coefficients, and each interval coefficient is associated with either a universal or an existential quantifier. The AE solution set [1–3] is defined by ∀∃-quantification. That is, a vector x is an AE solution if for every realization of ∀-coefficients there is a realization of ∃-coefficients such that x solves the corresponding system. Applications of this approach range from economic models, design problems to static control systems, among others.
Herein, we deal with the problem what properties must the coefficient matrix have in order that there is guaranteed an existence of an AE solution. Based on this motivation, we introduce a concept of AE regularity, which implies that the AE solution set is nonempty. We discuss characterization of AE regularity, and we also focus on various classes of matrices that are implicitly AE regular. Some of these classes are polynomially decidable, and therefore give an efficient way for checking AE regularity.

Keywords

Interval matrix, Interval computation, Linear equations, Forall-exists quantification.

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Application of interval linear algebra in data estimation

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There are various methods handling estimation of interval data. In our talk we focus mainly on least squares approach which can be solved by means of (interval) linear algebra. We show and discuss various methods of computing such estimation. We illustrate this approach on data obtained during children lung function diagnostics – multiple-breath washout procedure. Based on these examples, we discuss what kind of new insight into data interval estimation methods actually bring.

Keywords

Interval data, Estimation, Least squares, Children lung function diagnostics.

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Commutators and matrix functions

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Commutator of two matrices is defined by $[A, B] = AB - BA$ where $A, B \in M_n(C)$ and plays an important role in many branches of science. Our aim in this study is to get some results related to $[f(A), f(B)]$ by using properties of matrix commutators and some special matrix functions.

Keywords

Matrix commutators, Matrix functions, Norms.

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Estimation of parameters under the multilevel multivariate models

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The complexity of data has increased greatly over the last decade. Modern experimental techniques make it possible to collect and store multi-level multivariate data in almost all fields, in which several characteristics can be observed on more than one response variable at different locations, repeatedly over time, at different depths, etc. Such data can be presented in the form of a multi-index matrix (tensor) $\mathcal{Y}$. The third-order normally distributed tensor of observations, $\mathcal{Y} \in \mathbb{R}^{n\times p\times q}$ is discussed with the mean structured in the form of a generalized growth curve model, $[\![\mathcal{X}; \mathbf{A}, \mathbf{B}, \mathbf{C}]\!]$, with multiplication in all three directions of the third-order tensor $\mathcal{X}$ of unknown parameters by the known matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$. We present the estimation of an unknown tensor $\mathcal{X}$ of direct effects and a separable and doubly separable variance-covariance matrix.

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Linear spaces of symmetric nilpotent matrices

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In 1958 Gerstenhaber showed that if $\mathcal{L}$ is a subspace of the vector space of the square matrices of order n over some field $\mathbb{F}$, consisting of nilpotent matrices, and the field $\mathbb{F}$ is sufficiently large, then the maximal dimension of $\mathcal{L}$ is $\frac{n(n-1)}{2}$, and if this dimension is attained, then the space $\mathcal{L}$ is triangularizable. Linear spaces of symmetric matrices seem to be first studied by Meshulam in 1989 in view of the bound of their rank. Although it seems unnatural to ask when a linear space of symmetric matrices is made of nilpotents and when it is triangualar, we find a way to do so by going to an equivalent notion for symmetric matrices, i.e. persymmetric matrices. We develop a theory that enables us to prove extensions of some beautiful classical triangularizability results to the case of symmetric matrices. Not only the Gerstenhaber's result, but also Engel, Jacobson and Radjavi theorems can be extended. We also study maximal linear spaces of symmetric nilpotents of smaller dimension.

On some properties of weights matrices used in spatial analysis

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In a spatial analysis, contrary to a standard statistical analysis it is assumed that nearby georeferenced units are associated in some way. These spatial relationships are described very often by a spatial weights matrix. The matrix quantifies the spatial relationship and is usually constructed on the basis of the contiguity matrix.

The basis for most spatial models is an indicator of whether one region is a spatial neighbor of another, or equivalent one. The knowledge of this subject is presented by a square symmetric matrix C with the *(i,j)*-th element equal to 1 if regions i and j are the neighbors to one another, and zero if otherwise. By convention, the diagonal elements of this spatial neighbors matrix are set to zero.

There is a large number of ways to construct such a matrix (e.g. linear contiguity, rook contiguity, bishop contiguity, queen contiguity). The idea can be extended to second order measures of contiguity and further. Another approach is based on the distance between the analyzed objects and can be also expanded in several ways. In spatial analysis, use is made mostly of slightly transformed contiguity matrices, usually called spatial weights matrices.

The role of the weight matrix and the question how the spatial analysis is sensitive to its different choices is discussed by [1] and [2]. In the paper, we analyze algebraic properties of different types of weight matrices used for empirical data.

Keywords

Weight matrix, Contiguity matrix, Spatial analysis.

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Best unbiased estimates for parameters of three-level multivariate data with doubly exchangeable covariance structure

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There are analyzed the properties of the estimators of doubly exchangeable covariance structure for three-level data (m dimensional observation vector repeatedly measured at u locations and over v time points). This structure is an extension of the block compound symmetry covariance structure.
Under multivariate normality, the free-coordinate approach is used to obtain unbiased linear and quadratic estimates for the model parameters. Optimality of these estimators follows from sufficiency and completeness of their distributions. As unbiased estimators with minimal variance, they are consistent.

Keywords

Best unbiased estimates, Three-level multivariate data, Doubly exchangeable covariance structure.

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Determinants of interval matrices

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In our talk we would like to address determinants of interval matrices – tightest interval enclosing determinants of all real matrices contained an interval matrix. We show some results on complexity of computing and approximating such interval determinants. Then we introduce various methods based on preconditioning, Hadamard inequality, Gaussian elimination and Cramer rule that enables us to compute at least enclosures of interval determinants. For symmetric matrices we can make use of known enclosures of eigenvalues, that can help to obtain better enclosures of interval determinant. We also present numerical properties of mentioned methods.

Keywords

Determinant, Interval matrix, Complexity, Testing properties.

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Studying the inertia of an LCM matrix

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Let $S = \{x_1, x_2, \ldots, x_n\}$ be a set of distinct positive integers with $x_i \leq x_j \Rightarrow i \leq j$. The GCD matrix (S) of the set S is the $n \times n$ matrix with $\gcd(x_i, xj)$ as its ij entry. Similarly, the LCM matrix $[S]$ of the set S has $\operatorname{lcm}(x_i, x_j)$ as its ij entry. Both of these matrices were originally defined by H. J. S. Smith in his seminal paper [4] from the year 1876.

During the last 30 years both GCD and LCM matrices (as well as their various generalizations) have been investigated extensively in the literature. However, GCD matrices are in many ways easier to study than LCM matrices. For example, the GCD matrix (S) is positive definite for any set S whereas the LCM matrix $[S]$ is almost always indefinite and may be even singular. Very little is known about the inertia of the matrix $[S]$ in general. One can of course make some additional assumptions about the set S, but still the matrix $[S]$ remains quite hard to study. In 1992 Bourque and Ligh [1] conjectured that if the set S is GCD closed (that is, $\gcd(x_i, x_j) \in S$ for all $x_i, x_j \in S$), then the matrix $[S]$ is nonsingular. A few years later it was shown that this conjecture holds only for GCD closed sets with at most 7 elements, but not in general for larger sets (see [2] and [3]).

It turns out that if the set S is GCD closed, then the poset-theoretic semilattice structure of $(S, |)$ often alone determines the inertia of the LCM matrix $[S]$ completely. This is a bit surprising, since one could expect the exact values of the elements $x_i \in S$ to play a bigger role in this. In this presentation we are going to define a new lattice theoretic concept and use it to give an explanation to this mystery. We also show some examples how to determine the inertia of the matrix $[S]$ by looking only at the semilattice structure of $(S, |)$. At the same time we are able to give an elegant proof to the well-known result that the Bourque-Ligh conjecture holds for all except for one GCD closed set with at most 8 elements.

Keywords

LCM matrix, GCD matrix, Inertia, Eigenvalue, Bourque-Ligh conjecture, Möbius inversion.

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Confidence regions and tests for normal models with orthogonal block structure: pivot variables

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Models with Orthogonal Block Structure, OBS, have variance covariance matrices that are linear combinations of pairwise orthogonal projection matrices that add up to $\mathbf{I}_n$, that is

$$\mathbf{V}(\boldsymbol{\gamma}) = \sum_{j=1}^{m} \gamma_j \mathbf{Q}_j, \tag{1}$$

see [1] and [2]. These models continue to play an important part in the theory of randomized block designs and contain the models

$$\mathbf{Y} = \mathbf{X}_0 \boldsymbol{\beta} + \sum_{i=1}^{w} \mathbf{X}_i \mathbf{Z}_i, \tag{2}$$

where $\boldsymbol{\beta}$ is fixed and the $\mathbf{Z}_1, \ldots, \mathbf{Z}_w$ are independent, with null mean vectors and variance covariance matrices $\sigma_i^2 \mathbf{I}_{c_i}, i = 1, \ldots, w$, when the matrices $\mathbf{M}_i = \mathbf{X}_i \mathbf{X}_i^\top$ commute and $R([\mathbf{X}_1, \ldots, \mathbf{X}_w]) = \mathbb{R}^n$. We will assume normality to use pivot variables to obtain confidence regions and, through duality, test hypothesis both for:
- variance components $\gamma_1, \ldots, \gamma_m$ and $\sigma_1^2, \ldots, \sigma_w^2$;
- estimable functions $\psi = \mathbf{c}^\top \boldsymbol{\beta}$ and estimable vectors $\boldsymbol{\psi} = \mathbf{C}\boldsymbol{\beta}$.

In deriving confidence regions for the $\sigma_1^2, \ldots, \sigma_w^2$ and ψ we had to apply the Glivenko-Cantelli theorem and related results to samples of values of pivot variables. Moreover, for $\boldsymbol{\psi}$, we had to consider families of samples in order to adjust confidence ellipsoids using a technique similar to least square adjustment of linear regression.
We include a numerical application to the results of an grapevine experiment. This application is interesting in showing the good behavior of pairs of samples for the positive and negative parts of the $\sigma_i^2, i = 1, \ldots, w$. Then we show that we have $\sigma_i^2 = \sigma_i^{2^+} - \sigma_i^{2^-}$, with $\sigma_i^{2^+}$ and $\sigma_i^{2^-}$ linear combinations of the $\gamma_1, \ldots, \gamma_m$.

Keywords

Confidence regions, Pivot variables, UMVUE, Variance components.

Acknowledgement

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On projection of a positive definite matrix on a cone of non-negative definite Toeplitz matrices

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For a given space of Toeplitz matrices, the aim of this paper is to find the projection of a given positive definite matrix on the cone of non-negative definite Toeplitz matrices. [1] claims that such projection is equivalent to the projection on linear space of Toeplitz matrices. We show that not all projections preserve non-negative definiteness. Solution of that problem is projection on a cone of non-negative definite Toeplitz matrices; cf. [2]
In this talk we give methodology and the algorithm of the projection. We base on the properties of a cone of non-negative definite Toeplitz matrices.
This problem can be applied in statistics, for example in the estimation of unknown covariance structures under the multi-level multivariate models; cf. Cui et al. (2016).

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Jordan triple product homomorphisms on triangular matrices to and from dimension one

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A map $\Phi : \mathcal{M}_n(\mathbb{F}) \to \mathcal{M}_m(\mathbb{F})$ is a Jordan triple product (J.T.P.) homomorphism whenever $\Phi(ABA) = \Phi(A)\Phi(B)\Phi(A)$ for all $A, B \in \mathcal{M}_n(\mathbb{F})$.
In work in progress, we study J.T.P. homomorphisms on upper triangular matrices $\mathcal{T}_n(\mathbb{F})$. We characterize J.T.P. homomorphisms $\Phi : \mathcal{T}_n(\mathbb{C}) \to \mathbb{C}$ and J.T.P. homomorphisms $\Phi : \mathbb{F} \to \mathcal{T}_n(\mathbb{F})$ for $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$. In the later case we consider continuous maps and the implications of omitting the assumption of continuity.

Kronecker product approximation via entropy loss function

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The aim of this talk is to determine the best approximation of a positive definite symmetric matrix $\mathbf{\Omega}$ of order n by $\mathbf{\Psi} \otimes \mathbf{\Sigma}$, where square matrices $\mathbf{\Psi}$ and $\mathbf{\Sigma}$ are arbitrary (unstructured) or one of them, say $\mathbf{\Psi}$, can be structured as compound symmetry (CS) correlation, i.e., $(1-\varrho)\mathbf{I}+\varrho\mathbf{1}\mathbf{1}^\top$, or autoregression of order one (AR(1))) correlation, i.e., $\sum_{i=0} \varrho^i \left(\mathbf{C}^i + \mathbf{C}^{i\top}\right)$ with $\mathbf{C} = (c_{ij})$, and $c_{ij} = 1$ if $j - i = 1$ and 0 otherwise. The best approximation means here that the entropy loss function (cf. [1])

$$f(\mathbf{\Omega}, \mathbf{\Psi} \otimes \mathbf{\Sigma}) = \operatorname{tr}\left(\mathbf{\Omega}^{-1}(\mathbf{\Psi} \otimes \mathbf{\Sigma})\right) - \ln\left|\mathbf{\Omega}^{-1}(\mathbf{\Psi} \otimes \mathbf{\Sigma})\right| - n$$

is minimized with respect to $\mathbf{\Psi} \otimes \mathbf{\Sigma}$, where $\mathbf{\Psi}$ is unstructured or structured as CS or AR(1).

We show that for a given $\mathbf{\Omega}$ and positive definite component of $\mathbf{\Psi} \otimes \mathbf{\Sigma}$, say $\mathbf{\Sigma}$, the best approximation is obtained for positive definite $\mathbf{\Psi}$.

Presented results can be widely used in multivariate statistics, for example for regularizing the covariance structure of a given covariance matrix, for determining the estimators of covariance structure or for testing hypotheses about the covariance structures.

Keywords

Kronecker product, Approximation, Entropy loss function.

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Normal approximations for *vec*, trace and determinant of noncentral Wishart matrices

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Wishart matrices play an important role in normal multivariate statistical analysis. In this work we present an alternative approach which has been already used for normal vectors and is now applied to Wishart matrices, considering their *vec*, trace and determinant. The normal approximations we present hold when the norm of the non centrality parameters diverges to $+\infty$. Thus we have an attraction to the normal model, for increasing predominance of non centrality and not for increasing sample dimensions. Starting with the well behaved central matrices, and after going through the heavy noncentral Wishart matrices we obtain very convenient limit distributions when, as stated above, non centrality increases. Moreover, simulations showed that the threshold for the limit normal distributions is quite acceptable.

Keywords

Asymptotic linearity, Limit normality, Noncentral Wishart distributions, *vec*, Trace, Determinant.

Acknowledgement

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Neglecting non-diagonalizable matrices in social sciences

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In social sciences, many transition processes are described by Markov models[1]. Markov chains are characterized by stochastic matrices. In this paper, the interest lies with the non-diagonalizable stochastic matrices. We will explicitly show that it is possible for every non-diagonalizable stochastic 3×3 matrix to be perturbed into a diagonalizable stochastic matrix with real eigenvalues arbitrarily close to the original eigenvalues. This is done by using an additive perturbation[3]. This is based on the denseness of diagonalizable matrices in the set of stochastic matrices[2]. Moreover, every non-diagonalizable stochastic 3×3 matrix can be perturbed into a diagonalizable stochastic 3×3 matrix such that the principal (left and right) eigenspaces of the original matrix and the perturbed matrix coincide. An algorithm is presented to determine the perturbation matrix. Finally, a theorem is proved which shows that there are even more parallels between the original matrix and the perturbed matrix.

Keywords

Markov models, Stochastic matrices, Non-diagonalizable matrices, Perturbation theory.

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More about linear sufficiency in the linear mixed model

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A linear statistic $\mathbf{Fy}$ is called linearly sufficient, or shortly BLUE-sufficient, for the estimable parametric function of $\mathbf{K}\beta$ under the linear model $M = \{\mathbf{y}, \mathbf{X}\beta, \mathbf{V}\}$ if there exists a matrix $\mathbf{A}$ such that $\mathbf{AFy}$ is the best linear unbiased estimator, BLUE, for $\mathbf{K}\beta$. Similarly, $\mathbf{Fy}$ is called linearly prediction sufficient, or shortly BLUP-sufficient, for the new "future" observation $\mathbf{y}_*$, say, if there exists a matrix $\mathbf{A}$ such that $\mathbf{AFy}$ is the best linear unbiased predictor, BLUP, for $\mathbf{y}_*$. The new observation $\mathbf{y}_*$ is satisfying $\mathbf{y}_* = \mathbf{X}_*\beta + \mathbf{e}_*$, where $\mathbf{X}_*\beta$ is estimable, and the covariance matrix between $\mathbf{e}_*$ and $\mathbf{y}$ is known. Our purpose is to predict $\mathbf{y}_*$ on the basis of $\mathbf{y}$.

The concept of linear sufficiency was essentially introduced in early 1980s by [1,2]. In this paper/talk we generalize their results in the spirit of recent papers [3] and [4]. In particular, we pay attention to the linear sufficiency of $\mathbf{Fy}$ with respect to $\mathbf{y}_*$, $\mathbf{X}_*\beta$ and $\mathbf{e}_*$ and the mutual relations between these sufficiencies.

Keywords

BLUE, BLUP, Linear sufficiency, Linear mixed model.

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Robustness in the multivariate Gaussian distribution

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In this paper the methods developed by Magnus [1] are used to derive robust estimators of the variance of the estimated covariance matrix in a multivariate Gaussian distribution. In addition the profile likelihood for the correlation coefficient and partial correlation coefficients are derived. Using the methods developed by Royall and Tsou [2] robust versions of these likelihoods are developed.

Keywords

Robustness, Multivariate Gaussian distribution, Profile likelihood.

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Generalized Jacobi and Gauss-Seidel method for solving non-square linear systems

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In [5], authors considered non-square linear system with number of variables is more than that of equations and described a new iterative procedure along with a convergence analysis. Jacobi and Gauss-Seidel methods are most stationary iterative methods for finding an approximate solution to square linear systems. Using similar technique as in [5], Jacobi and Gauss-Seidel procedures for solving non-square linear system with number of variables is more than that of equations, are generalized. More specifically, Jacobi or Gauss-Seidel iterative methods are applied for the square part of the system and the iterative method described in [5] is applied for the non-square part of the system to obtain an approximate solution of the system. We also derive sufficient conditions for the convergence of such procedure. Finally, a procedure to obtain an exact solution of the system is provided. Numerical illustration has been given for the same, and to compare the procedure with the procedure available in [5].

Keywords

Iterative method, Jacobi, Gauss-Seidel, Convergence.

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A new method for determining the radius of regularity of parametric interval matrices

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The problem of determining the radius of regularity r^* of a parametric interval matrix is known to be NP-hard. In this paper a method for determining r^* is suggested, whose time complexity is not *a priori* exponential. The method is based on an equivalent transformation of the original problem to the problem of determining the real maximum magnitude (RMM) eigenvalue λ^* of an associated parametric generalised eigenvalue problem. The proposed method determines the regularity radius in polynomial if certain sign conditions are fulfilled. Otherwise, it produces upper bound $\overline{z}$ on r^*. Numerical examples of parametric interval matrices of large size illustrate the potential of the method.

Keywords

Parametric interval matrix, Regularity, Regularity radius.

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Infinite matrices and the Jordan form

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The talk concerns the concept of a Jordan canonical form of a matrix. As it is well-known if F is an algebraically closed field, then every square matrix over F is similar to its Jordan form. The aim of the presentation is to introduce an analogue of the Jordan form of a $\mathbb{N} \times \mathbb{N}$ matrix and sketch the proof of the theorem stating that for every upper triangular $\mathbb{N} \times \mathbb{N}$ matrix a there exists a column finite (i.e. possessing in each column only a finite number of nonzero entries) matrix x such that $x^{-1}ax$ is a generalized infinite Jordan matrix.

Keywords

Jordan canonical form, Infinite matrix, Infinite dimensional vector space.

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On circulant matrices with Ducci sequences and Fibonacci numbers

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A Ducci sequence generetad by $A = (a_1, a_1, \cdots, a_1) \in \mathbb{R}^n$ is the sequence $\left\{A, DA, D^2A, \cdots\right\}$where Ducci map $D: \ \mathbb{R}^n \to \mathbb{R}^n$ is defined by

$$DA = D(a_1, a_1, \cdots, a_1) = (|a_2 - a_1|, |a_3 - a_2|, \cdots, |a_n - a_1|).$$

In this study, we examine some properties of matrices C, DC, D^2C, where $C = (c_0, c_1, \cdots, c_{n-1})$ is a circulant matrix ,whose entries consist of Fibonacci numbers, and D denotes Ducci map.

Keywords

Circulant matrix, Ducci sequence, Fibonacci numbers.

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Immanant inequalities on correlation matrices and Littlewood-Richardson's correspondence

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The Littlewood-Richardson rule is one of the most important properties to describe the representation theory of the symmetric group, i.e. the coefficient of the product of Schur functions can be calculated in a combinatorial way using Young diagrams. In [3], it is also pointed out that immanants, which are special cases of generalized matrix functions labeled by Young diagrams, have the same rule.
One of the most famous open problems involving immanants is Lieb's permanental dominance conjecture ([2]), a sort of analogue of Schur's inequalities ([4]). In this talk, we analyze the correlation matrix $Y_n = (n/(n-1)\delta_{ij} - 1/(n-1))$, which conjecturally gives sharper bounds of the inequalities, where δ_{ij} is the Kronecker delta function. Motivated by Frenzen-Fischer's result ([1]), i.e. $\lim_{n\to\infty} \operatorname{per} Y_n = e/2$, we explore the limiting behavior of immanants through the limit shape of Young diagrams. The Littlewood-Richardson rule will be applied as the key lemma. We will introduce other related topics.

Keywords

Immanant, Symmetric group, Littlewood-Richardson rule.

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Sums of H-unitary matrices

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Let $H \in M_n$ be an nonsingular and Hermitian. A matrix A is said to be H-*unitary* if $A^*HA = H$. The set of H-unitary matrices forms a multiplicative group. However, the sum of H-unitary matrices need not be H-unitary. We discuss some previous results and show analogous or new properties for sums of H-unitary matrices. For example, we show that every matrix can be expressed as a sum of H-unitary matrices. We also characterize all matrices expressible as a sum of two H-unitary matrices.

Keywords

Sums, H-unitary, Indefinite linear algebra, Indefinite inner product.

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Root location of polynomials with totally nonnegative Hurwitz matrix

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For a given real polynomial

$$p(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n, \qquad a_0 > 0,$$

the $n \times n$ matrix $H_n(p) = (a_{2j-i})$ is called finite Hurwitz matrix, and the matrix $\mathcal{H}_\infty(p) = (a_{2j-i})_{i,j\in\mathbb{Z}}$ is the *infinite* Hurwitz matrix.
It is known [3,2] that the total positivity of the matrix $\mathcal{H}_\infty(p)$ is equivalent to stability of the polynomial $p(z)$ (roots in the open left half-plane), while the totally nonnegativity of [1,4] the finite Hurwitz matrix $H_n(p)$ does not imply stability of $p(z)$. In this talk, we completely describe root location of the polynomial $p(z)$ whose finite Hurwitz matrix $H_n(p)$ is totally nonnegative.

Keywords

Root location, Polynomials, Totally nonnegative matrices.

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Fixed effects estimation in two-variance components models

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In linear mixed models, it is well known that inference about the fixed effects parameters is not straightforward even under normality assumptions. Unless the model is balanced, in other words, unless the model matrices meet the necessary and sufficient conditions for the best linear unbiased estimator (BLUE) and ordinary least squares estimator (OLSE) to coincide, there is no closed form of the empirical version of the BLUE obtained by, say maximum likelihood method. For example, in case of a two-way analysis of variance model with random effects the situation is challenging since it is not obvious how to set up the test statistic so that inference can rely on the F-distribution. We have adapted the ideas suggested in, e.g., [3],[4], [1], and [5] and developed an explicit estimator of fixed effects in a mixed linear model with two variance components (see [7]) under rather general conditions. The new proposed estimator is based on a partition of the sampling space and on a re-sampled subvector from a linearly transformed residual vector. The newly proposed estimator can be considered as an alternative to the classic moment estimators. Generalizations of the suggested estimator are also investigated.

Keywords

Fixed and random effects, Estimation, Resampling.

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Applications of the Vandermonde matrix in statistics

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The Vandermonde matrix has a long history. In statistics the matrix appears, for example, in the design of experiments and in multivariate statistical analysis. In particular, in multivariate statistical analysis the determinant of the Vandermonde matrix plays a key role through its determinant which is easily expressable. A few well known relations and some less well known recursive relations will be presented.

Keywords

Vandermonde matrix, Wishart matrix, Recursive relations.

Tools for numerical inversion of the characteristic functions and their applications

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The exact statistical inference frequently leads to a non-standard probability distributions of the considered test statistics, see e.g. [1–4]. Here we consider simple methods and algorithms for combining characteristic functions of selected probability distributions and their numerical inversion to evaluate the associated CDF and/or PDF. The suggested methods have been implemented as *The Characteristic Functions Toolbox* in MATLAB and R programming environment for statistical computing, [5]. The applicability of the methods and algorithms will be illustrated by computing the exact (small sample) distribution of some well-known test statistics (e.g. the exact null-distribution of the Bartlett test statistic for testing homogeneity of variances), and/or the distribution of selected test statistics in multivariate statistical analysis (as e.g. the distribution of the Wilks Lambda statistic).

Keywords

Characteristic function, Numerical inversion, Exact statistical inference, MATLAB/R.

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Both residual errors accurate algorithm for inverting general tridiagonal matrices

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Even though in most problems involving matrix inverse the numerical computation of the actual inverse is usually not necessary (the problem may be reformulated to solve a corresponding system of linear equations or a corresponding matrix equation), there seems to exist no computational system or numerical library which would miss a subroutine for numerical computation of the matrix inverse.
When using such a subroutine one could expect to obtain the most accurate result possible. Unfortunately, all numerical algorithms (that are known to the authors) for computing the matrix inverse suffer a *curse* that the larger of the residual errors, $\|AX - I\|$ and $\|XA - I\|$ (X denotes the computed inverse of a matrix A), may grow as fast as $\mathrm{cond}^2(A)$, where $\mathrm{cond}(A)$ is the condition number of A (we assume that A is not a triangular matrix).
In our presentation, we present the algorithm for inverting general tridiagonal matrices that overcomes the above curse, i.e. it computes the inverse for which both residual errors grow linearly with $\mathrm{cond}(A)$. In addition, the proposed algorithm has the smallest possible asymptotic complexity for the considered problem.
The proposed method is based on careful selection of formulas for the elements of A^{-1}, which preserves all recursive properties resulting from the equations $AX = I = XA$. Extensive numerical tests confirm very good numerical properties of the algorithm.

Keywords

Matrix inversion, Tridiagonal matrix, Recursive algorithm, Numerical stability.

Problems of inference in a special multivariate linear model

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Simplified variance structures in multivariate linear models can substantially reduce number of 2nd order parameters, and thus required number of observations for a valid inference. At the same time, such structures are in many cases reasonable, because they are implied by the design of experiments. However, to take them into account is not an easy task. We will show some statistics for basic location testing in models with such a simplified parameter structure, especially multivariate repeated measures data model. The performance of several test will be compared. Methods will be demonstrated on real medical datasets.

Keywords

Multivariate linear model, Location test, Special variance structure.

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A Sub-Stiefel Procrustes problem

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In the talk we consider a Procrustes problem on the set of sub-Stiefel matrices of order n. For $n = 2$, this problem has arisen in computer vision to solve the surface unfolding problem considered in [2]-[4]. A sub-Stiefel matrix is a matrix that results from deleting simultaneously the last row and the last column of an orthogonal matrix.
An iterative algorithm for computing the solution of the sub-Stiefel Procrustes problem is proposed. For these purposes we investigate the properties of sub-Stiefel matrices. We also relate the sub-Stiefel Procrustes problem with the Stiefel Procrustes problem and compare it with the orthogonal Procrustes problem.
The talk is based on the paper [1].

Keywords

Procrustes problem, Sub-Stiefel matrix, Approximation of a matrix.

References

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Application of Jordan algebra and its inference in linear models

Roman Zmyślony

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Properties of Jordan algebra (Jordan 1934) or quadratic subspace (Seely, 1977; Zmyślony, 1980) will be discussed from the point of view of statistical applications to inference in univariate and multivariate normal models. Both estimation and testing hypotheses will be presented. Special cases for random effects model and blocked compound symmetric (BCS) covariance structure for doubly multivariate observations (m dimensional observation vector repeatedly measured over u locations or time points), which is a multivariate generalization of compound symmetry covariance structure for multivariate observations, was introduced by Rao (1945, 1953) while classifying genetically different groups, and then Arnold (1979) studied this BCS covariance structure while developing general linear model with exchangeable and jointly normally distributed error vectors. The test about covariance structure will be presented.

Keywords

Testing hypotheses, Estimation of parameters, Jordan algebra, Linear models.

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Part VII

Posters

On parallel sum of matrices

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The parallel sum of Hermitian semidefinite matrices shows up in many applied problems, like electric circuits, statistics, numerical methods, etc. I will introduce the main properties of this matrix operation, and its relations with matrix mean inequalities. The main problem I am interested in is to find explicit solutions of the matrix equation $\mathrm{A} : \mathrm{X} = \mathrm{B}$ where $:$ denotes the parallel sum operation. There are known sufficient conditions for the existence of a minimal solution, but in general there are many possible solutions which are not yet studied.

Keywords

Parallel sum, Hermitian semidefinite matrices.

Models for stochastic symmetric matrices

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In this work we study the matrices of a structured family of stochastic symmetric matrices. This matrices are all of the same order k and correspond to the treatments of a base design. The most interesting case is when the matrices in the family have a dominant first eigenvalue. We then study the action of the factors in, on the components of the first structure vector. We will consider briefly the models for these matrices and then we show how to carry out inference for the structured family.

Keywords

Structured families, Symmetric matrices, Transversal analysis.

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Part VIII

Jaroslav Zemánek in memoriam

Tribute to Yaroslav Zemánek

Natalia Bebiano

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I first met Yaroslav Zemánek when he decided to visit the Maths Department of the University of Coimbra circa 1996, following a visit to the South of Spain. He took a bus in Sevilha and travelled by Algarve and Alentejo. At the time the Center of Mathematical Research in the Department invited him to give a talk, and it was impressive the way he communicated his results and open problems.

He was delighted when I took him to visit Biblioteca Joanina and Sala dos Capelos, which are part of the university old buildings and Unesco World Heritage. Nevertheless, Doctor Zemánek became much more excited when speaking about the Marcus-Oliveira conjecture on the determinant of the sum of two normal matrices with prescribed eigenvalues, a problem that still remains open.

I met him again in other ocasions, namely at some WONRA meetings. I was always fascinated for the clarity and insights of his lectures, his broad knowledge and the clever ideas he always came up with. I consider him a source of inspiration, being always ready to discuss mathematical questions and share with everybody his vast horizons.

Stay in peace.

My meetings with Jaroslav Zemánek – the way I remember them

Andrzej Sołtysiak

Adam Mickiewicz University, Poznań, Poland

My first meeting with Jaroslav Zemánek took place in May 1975 during the "VI-th Seminar on Functional Analysis" in Štefanova (near Žilina). This was one of the conferences organized every year somewhere in Czechoslovakia by Professor Vlastimil Pták. From then on we have seen each other very often at the Institute of Mathematics of the Polish Academy of Sciences in Warsaw or at many conferences in Czechoslovakia, Poland, Germany, Canada, and Slovakia.

During my talk I am going to tell about mathematical problems Jaroslav was interested in at the beginning of His mathematical career, why He moved from Prague to Warsaw, and what was the content of His PhD thesis. I also say about His way of "doing" mathematics, what kind mathematics He liked and also what He disliked in it.

Mathematics – the borderless passion of Jaroslav Zemánek

Iwona Wróbel

Warsaw University of Technology, Poland

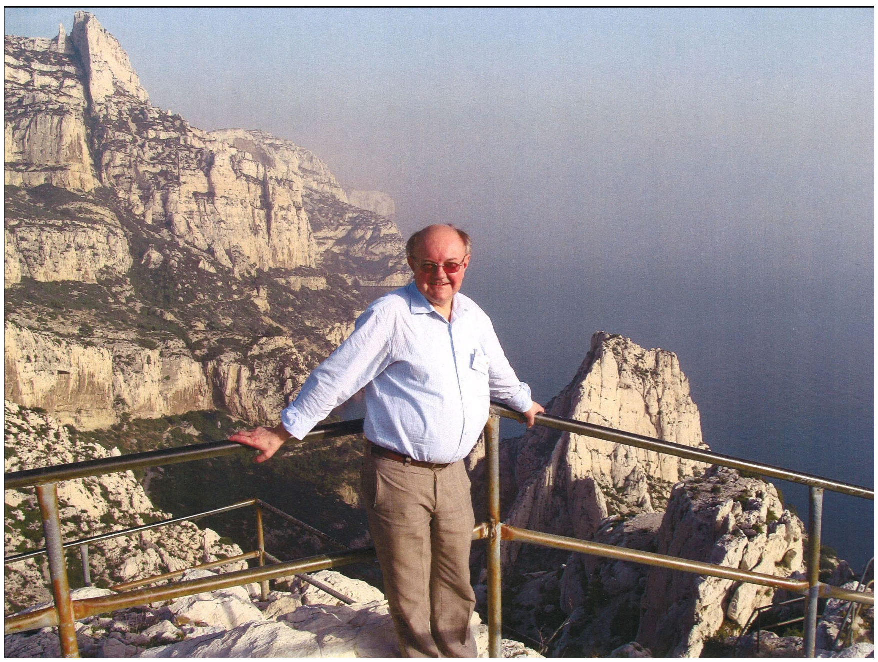
This talk is dedicated to professor Jaroslav Zemánek (03.09.1946 – 18.02.2017). A great mathematician, he first worked in Mathematical Institute of the Czechoslovak Academy of Sciences and then in Mathematical Institute of the Polish Academy of Sciences. He was an author of over 70 scientific papers, winner of several scientific awards (for instance Banach Prize of the Polish Mathematical Society, in 1987), member of editorial board of several journals, a mentor of 7 PhD students. I had an honour to be one of them.

Jaroslav Zemánek (03.09.1946 – 18.02.2017)

I first met Jaroslav Zemánek in 2001, when I was looking for someone patient enough to be a supervisor of my PhD thesis. After I finished it, we continued to have mathematical discussions and in time we became friends.

Jaroslav Zemánek and his beloved wife Barbara. Behind them the second love of his life – books. The photo was taken in Jaroslav and Barbara's apartment in Warsaw.

Mathematical interests of Jaroslav Zemánek were very broad, including mainly linear operators, Banach algebras, complex analysis, algebra, history of mathematics. And matrix theory, for instance topics related to localization of spectrum. I do not have enough knowledge to tell about everything. In this talk I will describe a couple of questions we were working on together, mostly related to matrices. Some of them remained open.
I will also briefly sketch biography of Jaroslav Zemánek, his interests, mathematical and other, mention some of the events he organized, his Operator Theory Seminars, the (somewhat unique) conference in Jurata, in 2010, and also tell about Jaroslav as I knew him.



Jaroslav Zemánek in his free time, during one of his many travels.

Matrices which Jaroslav Zemánek loved

Krystyna Ziętak

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I met Jaroslav Zemánek for the first time at the end of the 1980s or at beginning of the 1990s on a seminar in Warsaw. After the seminar all participants have decided to drink coffee and talk about recent conferences. Just when I had to leave for the railway station to return to Wrocław, Jaroslav Zemánek stood up and said "Why are you going? I would like to discuss matrices". In September 1991 or 1993 we went to Zakopane for a conference. From that time we had many opportunities to discuss matrices. In the talk I would like to say how Jaroslav Zemánek has inspired some my investigations, why we have no common paper, and what kind of man he was.

Keywords

Zero trace matrix, Moore-Penrose generalized inverse, Approximation of matrix.

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Part IX

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Index